Mathematical Crafts for Children: Beyond Scissors and Glue

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Abstract

Traditionally, craft activities have provided a tasteful introduction to mathematics for children: through the building of paper polyhedra, or the creation of patterns in string, children have often come to appreciate profound mathematical ideas. The next decade, however, is likely to see a tremendous expansion of accessible, powerful technology that will (or at least could) radically reshape and enrich the space of children's mathematical crafts. In this paper we describe a variety of technological innovations–ranging from software systems to fabrication devices to "smart" materials–that have the potential to become powerful tools for children's crafts. We discuss new directions for some traditional crafts–polyhedron-building, for example--and the evolution of never-before-seen crafts, such as building with computationally-enhanced construction kits or "printing out" customized complex mathematical forms.

1. Mathematical Crafts as an Element of Mathematics Education

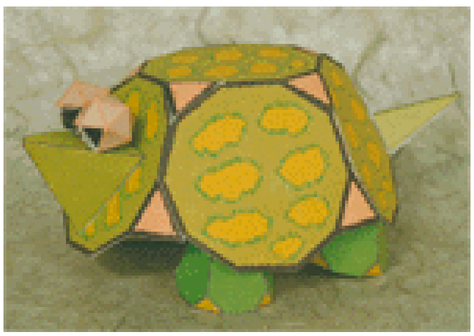
1.1. Children and the Design of Mathematical Objects. Mathematics education has a long–and, at its best, a wonderful–tradition of creating craft activities for children. Over the past century, students of mathematics have built mathematical models in paper, string, wax, and other materials; and these activities have served not only as an introduction to mathematical ideas (e.g., symmetry, solid geometry, and so forth), but also in subtler ways as beautiful and compelling invitations to the subject of mathematics. (See, for example, [1] for a beautiful and varied introduction to mathematical crafts, first published in 1951, and aimed at both students and teachers.)

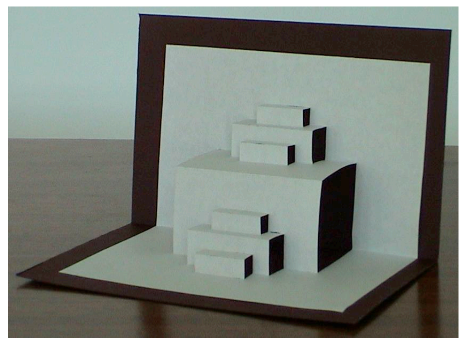
In the current educational landscape–populated as it is with simulation tools, video games, and virtual reality environments–traditional mathematical crafts have acquired something of a reputation for quaintness. Students still practice these crafts; but the activities themselves are often seen (both by teachers and students) as anachronistic. More importantly, the lack of connection between technological tools and mathematical crafts has unfortunate educational and creative consequences: by integrating new technology with mathematical crafts it should be possible to create not only more educationally powerful objects, but also more aesthetically compelling and individually expressive objects. In other words, mathematical crafts–as marvelous as they have been in the past–can now go still further beyond their traditional limitations.

1.2. What Technology Offers to the Practice of Children's Crafts. The purpose of this paper is to suggest ways in which relatively recent technologies can enhance the practice of mathematical crafts for children, and to illustrate those suggestions through examples drawn from projects in our laboratory at the University of Colorado. Briefly, we believe that the advent of computational media, affordable fabrication devices, and new (often "smart") materials has the potential to revolutionize the practice of mathematical crafts (and more broadly, mathematics education). (See [3] for additional discussion.) In this paper, we focus on several central themes of this larger argument: (i) the use of design software to allow children to go beyond "recipes" in the creation of mathematical objects, (ii) the use of fabrication devices to expand the range of children's craft materials, (iii) the way in which technologies impact the design and visual appeal of children's environments, and (iv) the use of embedded computation to expand the mathematical range and interest of craft activities themselves.

1.3. Outline of This Paper. The remainder of this paper expands upon the themes introduced in the previous paragraph. In the second section, we discuss each theme in turn, employing examples from our lab's projects as illustrations. In the third and final section, we step back from our particular examples and attempt to gauge how the renaissance of mathematical crafts might affect the larger practice of mathematics education.

2. Themes for Integrating Technology into Mathematical Crafts



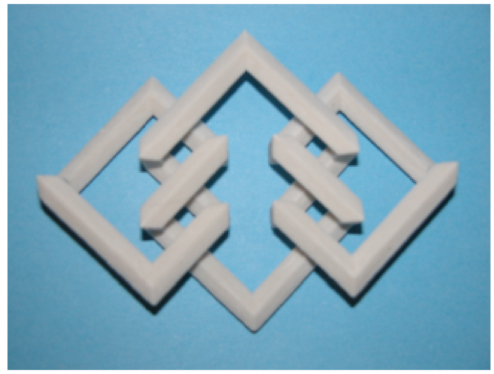


Figures 1 (left) and 2 (right). A paper polyhedral sculpture of a turtle constructed in HyperGami; a popup construction created using Popup Workshop.

2.1. Open-ended Design of Mathematical Objects. For the most part, traditional mathematical crafts have tended to follow a "recipe" model for construction: children might, for instance, be given a book of folding patterns with which to create particular polyhedral models, or they might be given instructions for creating a particular mathematical figure out of string. One distinct advantage of computational tools in this regard is that they can be conceived as *design applications*: that is, applications whose purpose is to provide children with general methods or techniques by which they can create all sorts of never-before-seen creations. In this sense, computers move children's crafts from a culture of "assignment-following" to one that is closer to "original mathematical-and-artistic creation".

Figures 1 and 2 illustrate this idea by showing constructions made with two programs developed in our lab. Figure 1 is a polyhedral sculpture created with HyperGami, a design application in which children can create models of an endless variety of polyhedral forms. In HyperGami, the student can begin a design session with a classical polyhedron (such as one of the Platonic or Archimedean solids); but she can then alter that shape by (among other possibilities) stretching it along a chosen axis, adding a new vertex outside a face, or slicing it into parts determined by a plane through three chosen points. Once a polyhedron is chosen, HyperGami will then attempt to "unfold" that shape into a two-dimensional folding net pattern which can the be decorated on the screen, printed on a color printer, and assembled.

Figure 2 is a popup form created in Popup Workshop, a program that allows children to view a template for a popup construction on the computer screen. The user places "cuts" of various types into the template, consistent with the practice of popup construction; as these cuts are placed, the user can view a 3D rendering of the resulting popup form as it opens and closes. Once a design has been completed, the student can decorate and print out the template; make cuts in the appropriate places marked on the printed form; and construct the desired physical popup.



Figures 3 (left) and 4 (right). A "sliceform" ellipsoid constructed in wood from laser-cut pieces; a plaster six-crossing knot printed out on a 3D printer.

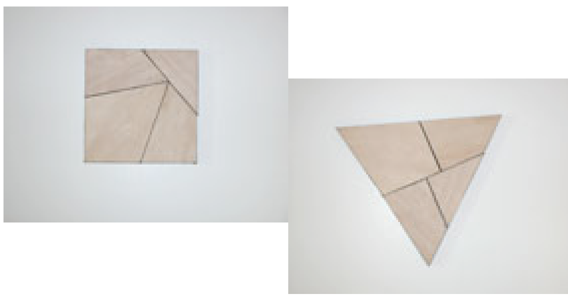
2.2. Fabrication Devices as Tools for Construction. One of the most important shifts in mathematical crafts is due to the increased power and affordability of fabrication devices–essentially, new sorts of output devices–that work in conjunction with computers. (Cf. the recently-published book by Gershenfeld [6] for an enthusiastic discussion.) There are a number of devices that are relevant to this theme, among which are: (i) laser cutters (which employ a laser to cut flat sheets of wood or plastic into desired shapes), (ii) 3D printers (which output 3-dimensional forms in plastic or plaster, among other possible materials), and (iii) computer-controlled sewing machines (which can embroider fabric according to computational control). In the realm of mathematical crafts, the presence of such devices permits us to view the computer as the control center for a "shop" that can produce mathematical models and objects that were hitherto beyond the range of any but the most advanced engineers and crafters. (See also the longer discussion in [4].)

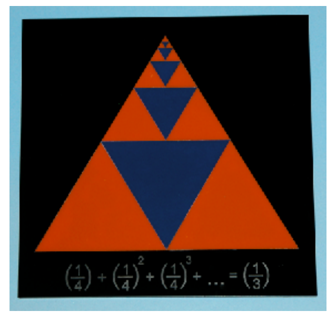
Figures 3 and 4 suggest the way in which novel fabrication devices can empower students of mathematics. Figure 3 shows a mathematical surface–an ellipsoid–constructed in wood using two orthogonal series of slotted planar pieces. (The technique for creating such "sliceform" surfaces is outlined in [1] and discussed at greater length in the beautiful book by Sharp [9].) Figure 4 shows a mathematical knot (based on an example in [7]) printed out in plaster on a 3D printer in our laboratory; this piece is an early suggestion of how 3D printers might be employed to create models of complex spatial forms.

Several points are worth emphasizing before leaving this topic. First, fabrication devices allow children to create or print out objects in an expanded range of materials (compared to traditional classroom crafts) such as wood, acrylic, and plastic. Second, the fact that new *materials* can be incorporated into student projects implies in turn that new *subject matter* can be incorporated into those projects. The projects in our laboratory include, for instance, a tool for printing out mechanical elements (such as gears) in wood: and the mathematics of custom-created gear trains thus suddenly becomes a subject of children's crafts. Likewise, the knot example of Figure 4 suggests the possibility of an entire "craft-based curriculum" in knot theory. Finally, the fact that finished, accurate mathematical models may now be printed out suggests that children can create customized objects with a "professional look" and with aesthetic appeal–a subject that we will expand upon in the following subsection.

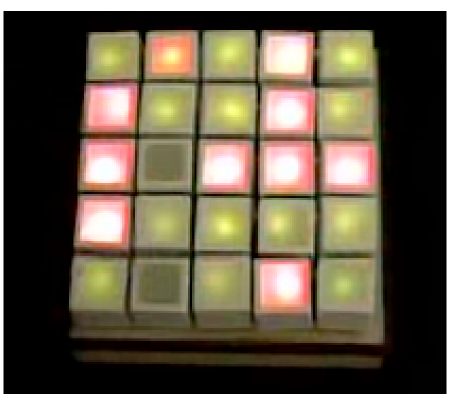
2.3. Display and the Customization of Children's Environments. One usually-unheralded aspect of mathematical crafts (particularly in comparison to purely "virtual", computer-based activities) is that the use of tangible materials permits children to create objects and artifacts that populate their physical space. In classrooms, it is not all that unusual to see mathematical models placed on shelves, hung from the ceiling, assembled into mobiles, and so forth. The contrast with computer-based activities here is telling: a mathematical game or simulation may be marvelous, but it remains (for the most part) "hidden" inside the computer; unlike the physical products of crafts, which are simply present and continuous in children's spaces, an educational computer program is invisible unless consciously accessed.

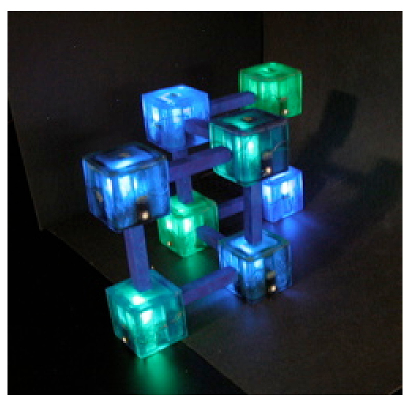
The technological tools and devices described in the previous paragraphs point toward a still-greater expansion of this "environmental" and "aesthetic" role of mathematical craft objects. In our own lab, we have seen how a growing collection of mathematical artifacts can turn a workspace into something closer to a personalized museum. Figures 5 and 6 are examples that suggest the uses of craft productions for demonstrations and displays. Figure 5 shows a hinged geometric dissection (in which a wooden square can be rearranged smoothly into an isosceles triangle [5]); this highly precise model was created with wooden pieces output from our lab's laser cutter. Figure 6 also shows a laser-cutter-produced artifact: here, brightly colored acrylic pieces have been cut to produce a pictorial display of a "proof without words" (taken from the delightful collection [8]). One aspect of Figure 6 worth noting is that this proof could be conveyed on paper (and in black-and-white); but the use of laser-cut acrylic turns the proof into something closer to a permanent piece of mathematical artwork that could be employed in a mathematics classroom. (See [2] for a lengthier discussion of the ideas of this subsection.)





Figures 5 (left) and 6 (right). Two views of a hinged dissection (in wood, constructed using a laser cutter); and, at right, an acrylic display of a "proof without words". (Constructions done by J. Blake.)





Figures 7 (left) and 8 (right). Embedded computers within construction pieces are used to create two-dimensional (left) and three-dimensional (right) "cellular automaton kits".

2.4. Mathematical Objects with Computationally-Specified Behaviors. The previous subsections have focused on using computers as design tools with which to specify or print mathematical objects and artifacts. Yet another way in which technology promises to change the practice of mathematical crafts is through the use of embedded computation. By placing computers inside craft objects themselves, we can make those objects programmable, and can endow them with qualities of interactivity and autonomous behavior. In turn, this enables new sorts of mathematical content to be brought into the sphere of craft activities.

Figures 7 and 8 provide illustrations of this theme. Both these figures show projects in our lab in which small computers (and LEDs) are placed within "building blocks". The Figure 7 example shows a two-dimensional array of programmed tiles; each tile contains its own, independently-programmable computer, and runs a program based on communication with its immediate neighbors. In Figure 7, the set of tiles has been programmed as a cellular automaton [10] to play Conway's famous "Game of Life". The three-dimensional set of blocks in Figure 8 likewise can run (3-dimensional) cellular automaton programs; each block contains its own microprocessor and can communicate with the blocks immediately linked to it. (See [2] for more detail on an earlier version of this project.)

3. Conclusion

The examples discussed in this paper are, we believe, early illustrations (or harbingers) of what we hope will be a larger sea change in the practice of mathematical crafts. The use of technology–design software, fabrication devices, embedded computation, and so forth–has the potential to spark a culture of "folk mathematics" among both children and adults. In this culture, we would hope to see a democratization of mathematical practice–rather than mathematics being some sort of rarefied and highly professionalized domain, mathematics could instead be the focus of a much broader and informal "hobbyist" audience. An energized tradition of mathematical crafts may thus impact not only children's lives (and minds) but those of adults as well.

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Acknowledgments

This work was funded in part by the National Science Foundation (awards no. EIA-0326054, and REC0125363), and by a gift from Mitsubishi Electric Research Laboratories (MERL) in Cambridge, Massachusetts. Susan Hendrix is the author of Popup Workshop; Nwanua Elumeze and Leah Buechley are the creators of the "cellular automaton" building block systems of section 2.4; Jenna Blake constructed the display pieces shown in section 2.3. We are grateful to Mark Gross, Michael Mills, Andee Rubin, Mitchel Resnick, Gerhard Fischer, Roy Pea, and Carol Strohecker for their ideas, influence, and conversation.