The Turtle in Space and on Surfaces, part II
Some Topics to Talk about

• Non-Euclidean Geometry -- the very idea
• Turtle moving on a sphere
• The idea of curvature density
• Intrinsic vs. extrinsic curvature
Euclid’s Postulates

1. It is possible to draw one and only one straight line from any point to any point.
2. From each end of a finite straight line it is possible to produce it continuously in a straight line by an amount greater than any assigned length.
3. It is possible to describe one and only one circle with any center and radius.
4. All right angles are equal to one another.
And one more...

5. If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
Some further tries

• All the points equidistant from a given straight line, on a given side of it, constitute a straight line. (Clavius, 1574)

• There exists at least one pair of equidistant straight lines.

• Through a given point not on a given straight line, and not on that straight line produced, no more than one parallel straight line can be drawn. (Playfair, late 18th century)
And a few more

- A rectangle exists. (Saccheri, 1733)
- A triangle with angle-sum 180 degrees exists. (Saccheri)
- It is possible to construct a triangle whose area is greater than any given area. (Gauss, 1799)
As for me, I have already made some progress in my work. However the path I have chosen does not lead at all to the goal which we seek [proving Postulate 5].... It seems rather to compel me to doubt the truth of geometry itself.

It is true that I have come upon much which by most people would be held to constitute a proof: but in my eyes it proves as good as nothing. For example, if one could show that a... triangle is possible, whose area would be greater than any given area, then I would be ready to prove the whole of geometry absolutely rigorously.

Most people would certainly let this stand as an Axiom; but I, no! It would, indeed be possible that the area might always remain below a certain limit, however far apart the three angular points of the triangle were taken.

– Karl Friedrich Gauss, in a letter to Farkas Bolyai, December 17, 1799