Fuzzy Logic; and an Intro to Probability

CSCI 5582, Fall 2007
Administrivia

- Problem Set 2 was sent out by email, and is up on the class website as well; due October 23 (hard copy for in-class students)
- To read over the weekend: Chapter 13 in the textbook
Fuzzy Logic

- Intermediate (between 0 and 1) assignments of truth, or set inclusion
- New definitions of AND, OR, and NOT
- Operators that correspond to “intensifying” or “modifying” set inclusion
What Might We Use Fuzzy Logic For?

- Representing “vague” or “indistinct” adjectives such as little, circular, rainy, etc.
- Representing prototypicality effects in cognitive science
Fuzzy Inference (If-Then) Rules

If temp is HIGH and speed is FAST
THEN STOP

If temp is MODERATE and speed is FAST
THEN SLOW-DOWN

If temp is LOW and speed is FAST
THEN NO-CHANGE
Fuzzy Logic as a Topic of Debate

• The “isn’t-this-just-\(x\)” argument (where \(x\) is, e.g., probability theory)

• Where do the numbers come from?
Probability: Some Preliminaries

• The possible events are represented by values of discrete (or occasionally, continuous) variables. Usually we describe the world with several variables. For instance, a weather description might be: (rainy, warm, windy)

• The set of atomic events is mutually exclusive and collectively exhaustive: (Heads1 Heads2), (Heads1 Tails2), (Tails1 Heads2), (Tails1 Tails2)

• Probability values are between 0 and 1. A probability of 1 is certainty; 0 is an impossibility.
Some More Preliminaries

- Set theoretic notions in probability: the probability of the union of (sets of) events; the probability of the intersection of sets of events; the probability of the complement of an event

Example:

\[ P(H_1) = P(H_1, T_1) + P(H_1, T_2) \]
\[ P(T_1) = P(H_1, T_1) + P(H_2, T_1) \]
\[ P(H_1 \text{ AND } T_1) = P(H_1, T_1) \]
\[ P(H_1 \text{ or } T_1) = P(H_1) + P(T_1) - P(H_1 \text{ AND } T_1) \]
\[ P(\text{NOT } H_1) = P(H_2, T_1) + P(H_2, T_2) \]
Conditional Probability

This can be read as “The probability that A is the case, given that we know that B is the case.”

\[ P(H_1|T_2) = \frac{P(H_1 \text{ AND } T_2)}{P(T_2)} \]

Or, in this case:

\[ \frac{P(H_1, T_2)}{[P(H_1, T_2) + P(H_2, T_2)]} \]

For a standard pair of coins, the probability of H1 is independent of (unaffected by) the probability of T2. So for this special case: \( P(H_1|T_2) = P(H_1) = 0.5 \)

Another way of representing independence:

\[ P(H_1 \text{ AND } T_2) = P(H_1) P(T_2) \]
A Gallery of Probability Judgment “Errors”

- Problem Framing
- The Conjunction Effect
- Ignoring Base Rates (Bayes' Theorem)
- Statistical Fluctuations
- Typicality Effects (and the "gambler's fallacy")
Problem Framing

Imagine that the United States is preparing for the outbreak of an unusual Asian disease that is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the program are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is a one-third probability that 600 people will be saved and a two-thirds probability that no people will be saved.

Which of the two programs would you favor?
Imagine that the United States is preparing for the outbreak of an unusual Asian disease that is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimates of the consequences of the program are as follows:

If Program C is adopted, 400 people will certainly die.

If Program D is adopted, there is a one-third probability that no one will die and a two-thirds probability that 600 people will die.

Which of the two programs would you favor?
First, we are offered a bonus of $300. Then, we are asked to choose between the two following possibilities:

A. To receive $100 for sure; or

B. To toss a coin. If we win the toss, we will get $200; if we lose, we receive nothing at all.
First, we are offered a bonus of $500. Then, we are asked to choose between the two following possibilities:

C. We are guaranteed to lose $100.

D. We toss a coin, and if we lose, we have to pay $200, but if we win, we don't have to pay anything.
The Conjunction Effect

Bill is 34 years old. He is intelligent, but unimaginative, compulsive, and generally lifeless. In school, he was strong in mathematics but weak in social studies and humanities.

- Bill is a doctor, and his hobby is playing poker.
- Bill is an architect.
- Bill is an accountant.
- Bill plays jazz for a hobby.
- Bill surfs for a hobby.
- Bill is a reporter.
- Bill is an accountant who plays jazz for a hobby.
- Bill climbs mountains for a hobby.
Considering Base Rates

Steve is very shy and withdrawn, invariably helpful, but with little interest in people or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail.

- Reporter
- Architect
- Farmer
- Librarian
- Biologist
- Taxi Driver
A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50 percent of all babies are boys. The exact percentage of baby boys, however, varies from day to day. Sometimes it may be higher than 50 percent, sometimes lower.

For a period of one year, each hospital recorded the days on which (more/less) than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?
Typicality Effects

Which sequence of coin tosses is more likely?

H H H H H H H H

H T T H T H T H

Memory Effects

Estimate the proportion of English words that begin with the letter "K" versus words that have a "K" in the third position.
The “Let’s Make a Deal” Problem

• Behind one door is a great prize. (The usual "great prize" on the old Let's Make a Deal show was a car.)
• The other two doors conceal a "Zonk" prize. (The usual "Zonk prize" on the old show was a goat or some other farm animal.)
• You get to choose one door.
• Once you have chosen, Monty Hall will open a "Zonk" door among the remaining two doors, and will ask "Do you want to change your choice to another door?"

Should you change your choice?
Let’s Make a Deal: The Manic Version
Bayes’ Rule

\[ P(H|E) = \frac{P(E|H) \times P(H)}{P(E)} \]
A clinical test, designed to diagnose a specific illness, comes out positive for a certain patient.

We are told that:

1. The test is 90 percent reliable. That is, if we give the test to 100 people who have the disease, 90 percent of the tests will come out positive; if we give the test to 100 people who do not have the disease, 90 percent will come out negative.

2. On average, this illness affects 1 percent of the population in the same age group as the patient.

What is the probability that this patient actually has the illness?
\[
P(\text{sick} \mid \text{pos}) = \frac{P(\text{pos} \mid \text{sick}) \cdot P(\text{sick})}{P(\text{pos})}
\]

\[
P(\text{pos} \mid \text{sick}) \cdot P(\text{sick})
= \frac{P(\text{pos} \mid \text{sick}) \cdot P(\text{sick}) + P(\text{pos} \mid \text{well}) \cdot P(\text{well})}{P(\text{pos} \mid \text{sick}) \cdot P(\text{sick}) + P(\text{pos} \mid \text{well}) \cdot P(\text{well})}
\]

\[
0.9 \cdot 0.01
= \frac{0.9 \cdot 0.01}{(0.9 \cdot 0.01) + (0.1 \cdot 0.99)}
= 0.083
\]