Logics: A Sampler

CSCI 5582, Fall 2007
What is Truth?

- Paris is the capital of France
- An elephant is a mammal.
- \(2 + 2 = 4\)
- It is sunny today.
- Lizzie Borden committed two murders.
- The Rockies will win the pennant in 2007.
- An urn is filled with 1000 ping-pong balls, 500 of which are white and 500 of which are red (thoroughly mixed). If we withdraw a single ball, it will be white.
- Colorless green ideas sleep furiously.
- An ostrich is a bird.
- My dog Rhombi (a terrier) is little.
Three Three-Valued Logics

- Kleene’s logic: $u$ means “unknown”
- Lukasiewicz’s logic: $i$ means “unknowable”
- Bochvar’s logic: $m$ means “meaningless”
Modal Logic

• We add the ideas of “necessity” and “possibility”
• $L_p$ means (roughly) “it is necessary that $p$”
• $M_p$ means (roughly) “it is possible that $p$”
Adding Modal Axioms to Propositional Logic

• The (weak) system T adds the following:
  IF (Lp) THEN p
  IF [L(IF p THEN q)]
  THEN [IF Lp THEN Lq]

• All tautologies are necessarily true

• We add a notion of “entailment”:
  p ENTAILS q (sometimes p --> q)
  means
  L (IF p THEN q)
Two More Modal Systems: S4 and S5

• In S4, we add the axiom:
  IF (Lp) THEN (L (Lp))

• In S5, we add the axiom:
  IF (Mp) THEN (L (Mp))
What Does All This Mean, Anyway?

- IF (Lp) THEN p
  “Things that must necessarily (logically?) be true are true.”
- IF (Lp) THEN (L (Lp))
  “Things that are necessarily true must be necessarily true.”
  Alternatively: “If I know that p, then I know that I know that p.”
- IF (Mp) THEN (L (Mp))
  “If p is possible, then it is always (necessarily) possible.”
Thinking of Modal Logics in Terms of “Possible Worlds”

• We represent all possible worlds as nodes in a graph linked by “accessibility”

• In this case, our new modal axioms can be thought of as characterizing the type of possible-world graph that we are thinking about.
Modal Axioms and Their Graphs

• IF (Lp) THEN p
  is characteristic of a reflexive graph
• IF (Lp) THEN (L (Lp))
  is characteristic of a transitive graph
• IF p THEN (L (Mp))
  is characteristic of a symmetric graph
• IF (Mp) THEN (L (Mp))
  is characteristic of a reflexive, transitive, symmetric graph
Deontic Logic: What Ought to Be

• We might treat L as the “ought” modality and M as the “permissible” modality:
  Lp means “it ought to be the case that p”
  Mp means “it is permissible that p is the case”

• If we think of the standard modal operators in these terms, we need to rethink our axioms as well. Consider the “weak” axiom:
  IF Lp THEN p
Fuzzy Logic

• Intermediate (between 0 and 1) assignments of truth, or set inclusion
• New definitions of AND, OR, and NOT
• Operators that correspond to “intensifying” or “modifying” set inclusion
• Fundamental concerns of (debates around?) fuzzy logic
Fuzzy Inference (If-Then) Rules

If temp is HIGH and speed is FAST
    THEN STOP

If temp is MODERATE and speed is FAST
    THEN SLOW-DOWN

If temp is LOW and speed is FAST
    THEN NO-CHANGE