First-Order Predicate Logic

CSCI 5582, Fall 2007
What Can’t We Do in FOPL?

• We can’t take anything back once we’ve asserted it.
• We can’t make statements about relations themselves (e.g., “Brother is a commutative relation”)
• We can’t distinguish between “types” of truth
• We can’t express degrees of belief
Some Sample Wumpus World Statements

FOR-ALL(s)
    Breezy(s) <--> THERE-EXISTS(p)[Adjacent(p,s) and Pit(p)]

FOR-ALL(x,y) (Wumpus(x) AND NOT(x = y)) --> NOT(Wumpus(y))

FOR-ALL(x,y)(Wumpus(x) AND Wumpus(y)) --> (x=y)

THERE-EXISTS(x) Wumpus(x)
Backward Chaining: the Basic Idea (Propositional Calculus)

Suppose we have a given knowledge base and wish to see whether we can prove a particular assertion (say, “Q”). Look to see whether Q is the consequent (“head”) of any Horn clause (e.g., P-->Q) and see if the body of that Horn clause has been asserted.

If not, continue with this process by seeing if the body of the Horn clause can itself be proven by backward chaining. (For instance, we may find a clause of the form (A AND B)---> P where both A and B have already been asserted.)
Redoing this idea for FOPC

- Arrange statements syntactically so that they are appropriate for backward chaining.
- Our statements now look like big IF-THEN statements (or rules, if you like), many with variable names.
- Run backward chaining, unifying a goal query with the head of any appropriate rule.
The First (Syntactic) Step

1) Eliminate occurrence of IMPLIES (and EQUIVALENT-TO)
2) Bring negation inside parentheses:
   NOT (FOR-ALL (x) P(x)) becomes
   THERE-EXISTS(x)(NOT P(x))
   NOT (THERE-EXISTS (x) Q(x)) becomes
   FOR-ALL(x) (NOT Q(x))
3) For every occurrence of a quantifier, provide the bound variable with its own unique name. (This prevents name conflicts when quantifiers are eventually "moved to the front."
More Syntax

4) Use Skolemization to replace existential quantifiers. The idea here is that an existential quantifier specifies a "choosing function" (that may depend on other variables):

   FOR-ALL (x) [THERE-EXISTS [y] Successor (y x)]

   becomes

   FOR-ALL (x) [Successor (Successor-of(x), x)]

5) We now have a sentence in which only universal quantifiers occur, and in which all variables have unique names. We can now move all these quantifiers to the left of the overall sentence without changing the meaning of the sentence.

6) Rewrite the sentence as a conjunction of disjunctions.

7) Rewrite as a set of separate sentences, each a disjunction.
Even More Syntax

8) Bring the negated terms to the left, and rewrite each sentence as an implication.

9) Within each sentence, rename variables to sentence-specific values. This is in preparation for...

10) (For brevity) Eliminate the universal quantifiers.
Even More Syntax

8) Bring the negated terms to the left, and rewrite each sentence as an implication.

9) Within each sentence, rename variables to sentence-specific values. This is in preparation for...

10) (For brevity) Eliminate the universal quantifiers.
A Tiny Example

Father-of(x, y) AND Father-of (y, z) --> Grandfather-of (x, z)
Father-of (John-of-Gaunt, Henry4)
Father-of (Henry4, Henry5)

A query might look like:
Grandfather-of (?Who, Henry5)
Unify the head of our rule with the query:
[?X1 bound to ?Who, ?Z1 bound to Henry5]
With this binding in effect, unify the body of the rule.
So we need to unify
Father-of (?Y1, Henry5) with available statements:
[?X1 bound to ?Who, ?Y1 bound to Henry4, ?Z1 bound to Henry5]
And we need to unify:
Father-of (?X1, ?Y1) with available statements:
[?X1 bound to ?Who bound to John-of-Gaunt,?Y1 bound to Henry4, ?Z1 bound to Henry5]
Backward Chaining and Resolution

• Resolution is more general; but the backward chaining technique just shown is a special case of resolution.
• Resolution is at heart a search strategy: what should we resolve with what?
• Natural strategies for choosing candidates for resolution: “set-of-support” strategy, input resolution strategy
What is Truth?

• Paris is the capital of France
• An elephant is a mammal.
• $2 + 2 = 4$
• It is sunny today
• Lizzie Borden committed two murders.
• The Yankees will win the World Series in 2008.
• An urn is filled with 1000 ping-pong balls, 500 of which are white and 500 of which are red (thoroughly mixed). If we withdraw a single ball, it will be white.
• Colorless green ideas sleep furiously.
• An ostrich is a bird.
• My dog Rhombi (a terrier) is little.
Three Three-Valued Logics

• Kleene’s logic: $u$ means “unknown”
• Lukasiewicz’s logic: $i$ means “unknowable”
• Bochvar’s logic: $m$ means “meaningless”